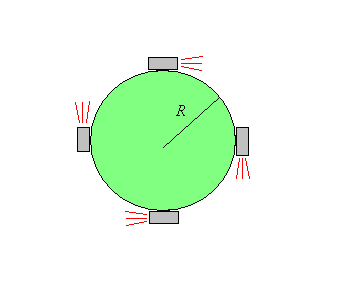
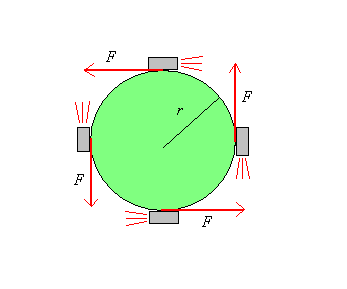
**Problem 8.33**

To get a flat uniform cylindrical satellite spinning at the correct rate, engineers fire four tangential rockets as shown. If the satellite has a mass of 3600kg and a radius of 4m, what is the required steady force of each rocket if the satellite is to reach 32 rpm in 5 min?



**Solution**

Let’s draw the forces that each rocket exerts on the satellite.



We need to adjust the force, F, so that the angular acceleration of the satellite is:

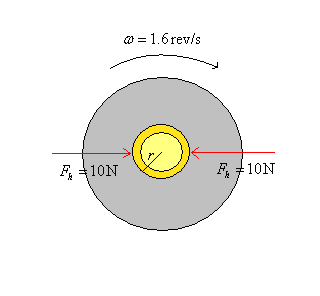


Therefore, according to N2L for rotation we will have,



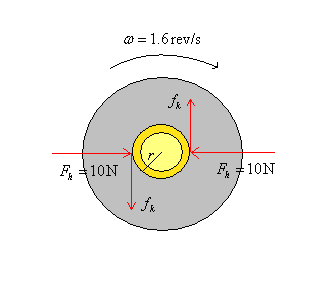
**Problem 8.30 (sort of – I’m changing the problem a little bit)**

A potter is shaping a bowl on a potter’s wheel rotating at constant angular speed. The initial angular velocity of the wheel is 1.6 rev/s, and the moment of inertia of the wheel and the bowl is I = 0.11 kg·m2. Suppose she exerts a force of 10 N on the bowl, with each hand, and that the coefficient of kinetic friction between her hands and the bowl is 0.15. (a) how large is the torque on the wheel, if the diameter of the bowl is 12cm? (b) how long would it take for the potter’s wheel to stop if the only torque acting on it is due to the potter’s hand? (a rough picture is given below – the cup is in yellow, connected to the potter’s wheel in grey).



**Solution**

When you push against the edges of the cup, the resulting friction will produce a torque on the cup. The friction force will be given by *f*­k = μkFh (because Fh functions as a ‘normal’ force here). The friction forces are displayed below:



So the net torque will be:



To determine how long it will take the wheel to stop, we will need to know the angular acceleration of the wheel. From N2L we have,

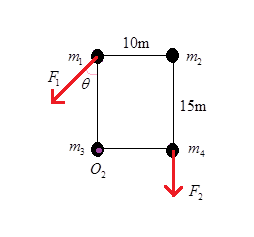


And now we will use the rotational kinematics equation, to determine when it stops. It will stop when ω = 0, and so we solve for t



(the negative sign on the 1.6 rev/s comes from the fact that the wheel is spinning clockwise).

5b. If you exert the forces F1 = 30N (θ = 20°), and F2 = 20N as shown below, what will be the object’s angular acceleration about the axis O2?



The torque exerted by **F**1 is:



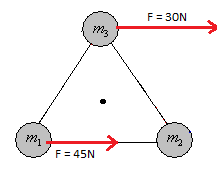
The other torque is:



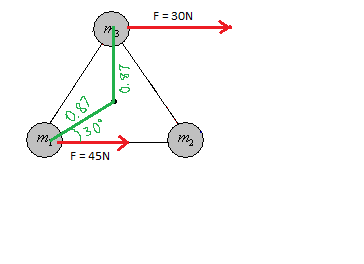
Filling this into N2L for rotation we have:



Question 2. Suppose the moment of inertia about this point were I = 4.5 kg∙m2. If we exert the folowing forces on the triangle, what will be its angular acceleration? Be sure to indicate whether it would be clockwise or counter-clockwise.



Drawing in the radii again:



The net torque is:



and so the angular acceleration would be:



So it will accelerate CW at rate of 1.44 rad/s2.

9. Consider a helicopter blade that is 6m long and has a mass M = 100kg. The motor which spins the blade must accelerate it up to a final angular velocity of ω = 140 revolutions/minute in 5s. What angular acceleration in rad/s2 does the helicoptor blade have? What torque must the motor provide?

First we have to convert ω to rad/s. So ω = 140 ∙ 2π / 60 = 14.6 rad/s. And the acceleration is therefore α = dω/dt = (14.6 rad/s – 0 rad/s)/(5s) = 2.92 rad/s2. Now from N2L we have:



**Problem 1.**

Suppose you pushing a child on a merry-go-round. It takes 12s for you to get it up to its final rotational velocity of 1.3 rev/s. If the diameter of the merry-go-round is 5m, what is the speed of the child in m/s?

Speed is given by:



**Problem 2.**

In the problem above, suppose that to get the merry-go-round up to this rotational velocity, you exert some average force tangential to its circumference during the 12s. What is this average force? You can treat the merry-go-round as a hoop, and take its mass to be 180kg. And you can take the mass of the child to be 45kg.

We can use the torque equation. First, the moment of inertial of the entire thing is I = mchild∙r2 + mmerry-go-round∙r2 = (225)∙(2.5)2 = 1406 kg∙m2. The acceleration (angular) is α = Δω/Δt = [1.3 rev/s]/12s = [(1.3)∙(2π) rad/s]/12s = 0.68 rad/s2. So then we have:



**Problem 3.**

In the problem above, suppose you now let go of the merry-go-round and it slowly comes to rest in 17 revolutions. What is its angular acceleration?

First let’s calculate the final θ = -17(2π) = -106.8 rad. And the initial angular velocity is ω = -1.3∙2π rad/s = -8.17 rad/s. We have the equations:



And secondly,



Plugging this into the top equation yields:



**Problem 6**

Consider a bicycle wheel set spinning at rate ω = 10π rad/s. The wheel has a mass M = 5.8kg, and radius r = 0.35m. If there is friction in the bearings that exerts a counter-torque τFriction = 1.8 N∙m, how long will it take for the wheel to come to rest? What angle will it have rotated through in this time?

Using N2L we have:



To figure out when it stops we can use the kinematics equation:

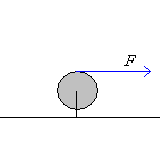


Note the (-) sign on the 10π is b/c I assume the wheel is initially rotating clockwise, and in any event, if α is to represent a slowing down of ω, then α and ω must have opposite signs. Next we want the angular displacement. This is:



**Example: Rotating disk**

Suppose that a grindstone, which we’ll approximate as a solid disk, is initially at rest. Then suppose we exert a constant force of 25N tangent to the edge of the grindstone (as illustrated below). What will be its initial angular acceleration? After the grindstone completes 10 revolutions, what will be its angular velocity? Assume the grindstone has a mass of 5kg, and a radius of 50cm.



To determine the initial angular acceleration of the grindstone, we will use N2L for rotation. Note that our axis of rotation we’ll take to be the center of the disk.



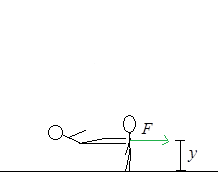
So the angular acceleration will be -20 rad/s2, i.e., in the CW direction. Now, after 10 revolutions, we can determine its angular velocity. Use the following equation,



The appropriate answer is -50.1 rad/s, since it will be rotating CW. Note that for Δθ we used -10·2π. We used the negative sign because it was rotating CW.

**Problem 6**

Batroc delivers a kick to CA’s chest with a force F = 300N, at height y = 1.5m, causing CA to rotate clockwise. Assuming CA’s feet remain fixed, what is CA’s angular acceleration? You may treat CA as a board with mass M = 100kg, and height L = 1.9m.



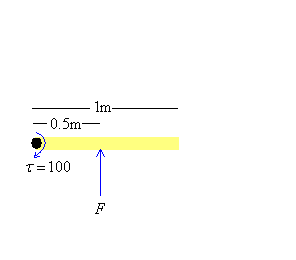
We have:



**Example: Pushing open a door**

Suppose you’re trying to push open a door of mass 50kg, width 1m, and height 2.5m. If the hinges exert a counter torque of τhinge = -100N·m**,** then what force would you have to apply to move the door at a distance of 0.5m from the hinge?

The situation is illustrated, top-down, below:



We can apply N2L of rotation to the door, which we take to be our system.



**Example: Pushing open a door still**

Suppose that we’re pushing the door with a force of 220N, at the same location. How long will it take to push it through an angle of 90 degrees?

Again, we’ll apply N2L law to the door. And this time, we’ll use the fact that I = (1/3)ML2, where L is the length (1m) of the door. So we have,

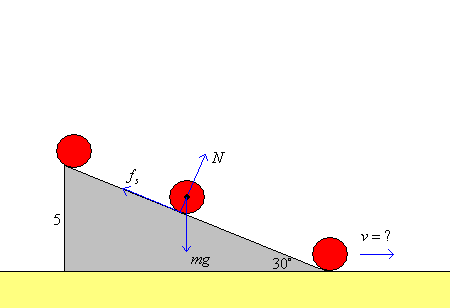


And so the time it takes to go through an angle of 90 degrees = π/2 radians is given through the following equation.



**Example: Ball rolling down a hill**

Let us calculate how long it will take a basketball to roll down a hill of height 5m and incline 30˚. Suppose the ball has a mass 0.5kg, and radius 10cm.



We will take the ball to be our system. We will apply the force and torque equations to the ball. Note that we do not know what fs is. We do not know that it is μsN since no information in the problem indicates that friction is at its maximum value. So we will merely assume some arbitrary value. As for the forces, we have,



and for the torque we have the following. We will examine the torques about the center of the sphere. In that case,



where we note that the torque supplied by N is 0 because there is no component of it perpendicular to the moment arm R. Also, the angular accceleartion is clockwise (in the negative z direction). Now we don’t know fs, nor a, nor α. So we have 3 unknowns, and 2 equations which isn’t good. We need one more. The remaining equation is the one which connects a and α. Consider that a, the acceleration of the ball down the hill, a, is also at, the tangential acceleration of the rim of the sphere. Therefore, a = at = αR, as previously discussed. Therefore α = a/R.



So then, solving for fs and plugging into the first equation gives,



Note how a doesn’t depend on m or on R, so all spheres regardless of mass or radius will roll down at the same time – whether they be golf balls or basketballs. The velocity it will have at the bottom of the ramp will be:

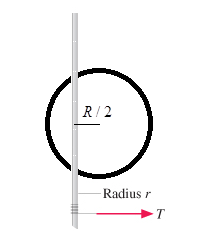


and the amount of time it will take to get there is:



**Problem 6**

A hoop of mass M and radius R is rigidly attached to a thin rod of radius r that passes through the disk at a distance R/2 from the center. A string is wrapped around the rod and pulls with a tension T. What angle will the disk have rotated through in time t?



We have:

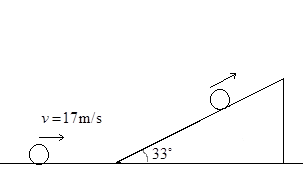


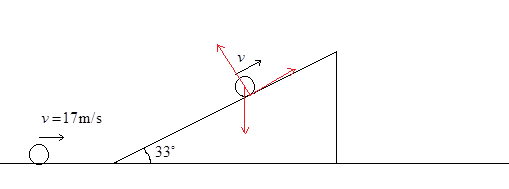
and so the angle rotated through after time t is:



**Problem 6**

Suppose you roll a disk up a slope with initial velocity v = 17m/s. When will it come to rest?





I’ve drawn forces. Now let’s see what the torque equation says,



Now use relationship a = -αr to write this in terms of the acceleration of the ball up the plane,



Now let’s use N2L in x direction (up the slope) to figure out *a*…

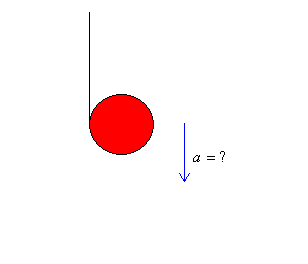


So then to figure out when it stops we use:

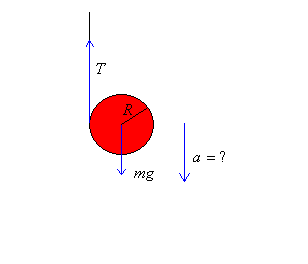


**Example: Yo-yo**

Suppose you have a 0.25kg yo-yo shaped like a disk with diameter 5cm. If you let it unwind down the string, what will be its acceleration?



to find out, we draw the forces acting on it and use the force/torque equations.



Orienting our y-axis downward, the force/torque equations yield,



We will apply the torque equation about the center of the disk. Then the torque equation yields,



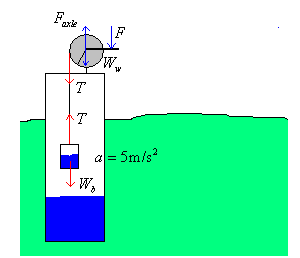
Now plug this equation into equation 1,



so we find the acceleration to be 2/3 that of gravity. Note how this is independent of the mass of the yo-yo.

**Example: Pulling bucket up from well**

Consider the following problem where we are pulling a bucket up from well using a crank. Suppose the bucket of water has a mass of 10kg. The wheel we’ll approximate as a disk with a mass of 5kg, and radius 25cm. To reel the bucket up, we exert a force, F, on the handle connected to the wheel. Now, what torque must this force exert on the handle to pull the bucket up with an acceleration of 5m/s2? Assume that the axle is frictionless.



Write out N2L (rotation) for the wheel,



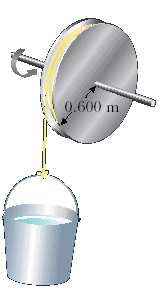
Now use N2L for translation of the bucket,



Solve for T and plug into the first equation,



11. A 1.2 kg cylindrical reel with a radius of 0.45 m and a frictionless axle, starts from rest and speeds up uniformly as a 4 kg bucket falls into a well, making a light rope unwind from the reel. The bucket starts from rest and falls for 10 s. What is the bucket’s speed at this time?



Applying N2L to the reel we have:



and applying it to the bucket we have:



Setting these two equal and solving for the acceleration we get:

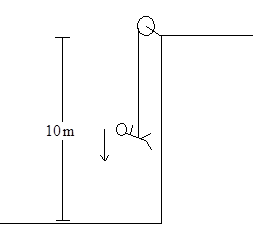


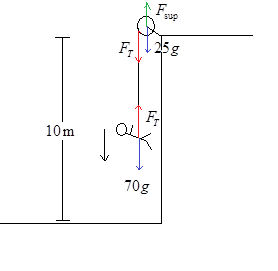
and so the speed of the bucket after 10s is…



**Problem 3**

Suppose you drop off a rock wall, your rope attached to a pulley, shaped like a disk. As you fall, the rope will spin the disk (like a fish pulling on the reel of a fishing pole) which will in turn slow your rate of decent. Suppose the mass of the disk is M = 100kg, with radius R = 0.5m. And suppose your mass is m = 70kg. Draw the tension and gravitational forces acting on you, and the tension, gravity and support force acting on the disk. Note that the tension acting on you is the same as the tension acting on the disk.





**Problem 4**

Referring to the problem above, write down N2L in the y-direction (ΣFy = may) for the person. And write down N2L of rotation (Στ = Iα) for the disk. You should have 3 unknowns in your equations: the tension FT in the rope, the acceleration of the person ay, and the angular acceleration of the disk α.



and,



**Problem 5**

The person’s acceleration ay is related to the disks angular acceleration α. What is this equation (called the rolling condition)? Note we used this equation when analyzing the rolling snow-ball problem. Should there be a minus sign in this equation? Why or why not (consider the expected sign of the ay vs. the expected sign of α)?



Negative sign is because ay is negative, whereas α is positive.

**Problem 6**

Substitute the equation in problem 5 into the Στ = Iα equation in problem 4. And now solve for ay.

Now we have:



Plugging this into the force equation and solving for ay we get:



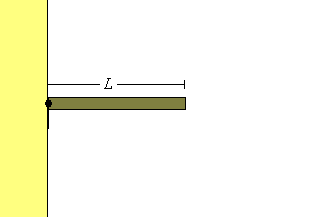
**Problem 7**

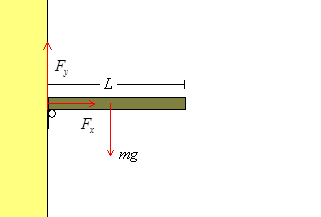
If the person drops from rest from a height of h = 10m, how long will it take to hit (hopefully softly) the floor?

We can use the y-equation:



4. A uniform rod of mass M and length 95cm can pivot freely about a hinge attached to a wall. The rod is held horizontally and then released. At the moment of release, determine the magnitude of the angular acceleration of the rod (in rad/s2). Note the wall will again exert forces in both the x and y directions on the rod. (α = -15.5 rad/s2)

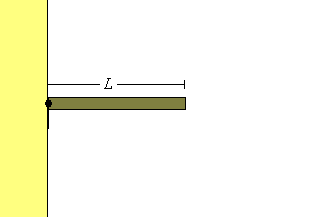




First we’ll draw the forces acting on the rod, and then apply N2L for rotation,



**Question 11**. A uniform rod of mass M and length ℓ = 1m can pivot freely about a hinge attached to a wall. The rod is held horizontally and then released. Assuming its angular acceleration is always equal to its initial angular acceleration, estimate when the rod will hit the wall.



The initial angular acceleration will be given by:



So it will hit the wall when,



**Question 6**. A girl exerts a 115 N force tangent to a solid sphere. If the sphere has a mass m = 900kg and radius R = 72cm, how long until it has completed one revolution?



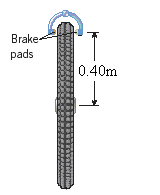
Torque equation…



and now, kinematics equation to get the time…



5. A stationary bicycle is raised off the ground, and its front wheel (*m* = 5 kg) is rotating at an angular velocity of 10 rad/s. The front brake is then applied for 2s, and the wheel slows down a stop. Assume that all the mass of the wheel is concentrated in the rim, the radius of which is 0.40m. The coefficient of kinetic friction between each brake pad and the rim is μk = 1.30. What is the magnitude of the normal force (in Newtons) that *each* brake pad applies to the rim? (FN = 3.85 N)



The two friction forces exert a torque which slows the wheel down. Applying N2L for rotation we have,



Now α can be obtained via:

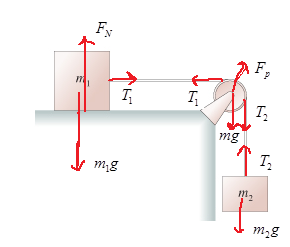


Plugging this in…



**Problem 5.**

Block m1 = 1kg is attached to block m2 = 2kg via string draped over a pulley with moment of inertia I = 0.03 kg∙m2 and radius R = 10cm. (a) what will be the acceleration of the blocks? (b) What will be the tension in the two ropes?

****

Free body diagram is drawn. Then we apply N2L on block 1, the pulley, and block 2, using axes that go to the right and down:



Now solve for the T’s and plug both into the middle equation:



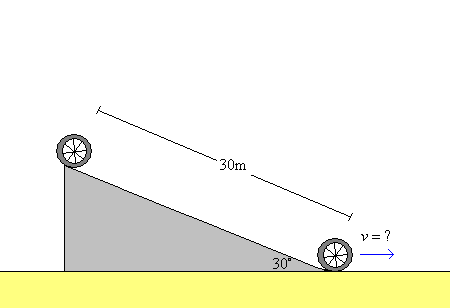
Now we can go back and get the tension in the first string



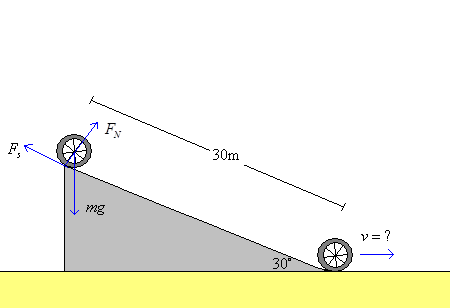
and the second string:



6. Suppose a bicycle tire (m = 5kg and radius r = 40cm) rolls down a hill 30m long with an angle of incline θ = 30˚. What will be its speed (in m/s) at the bottom of the hill? Note you can treat the tire as a hoop. (v = 12.1 m/s)



Drawing the forces…



We will use N2L for rotation to determine the acceleration, and then kinematics equations to determine the velocity. So N2L for rotation says,



Now relate the angular acceleration to the linear acceleration using, a = -αr (because a is positive while α is negative), so we can write:



Now apply N2L in the x-direction. We have,



and now to determine the speed at the bottom of the hill we first determine how long it will take for the wheel to roll to the bottom. This can be found using the x-equation.



and now can get the velocity…

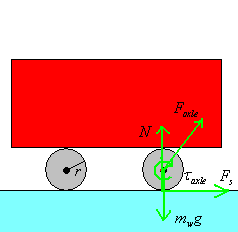


**Example: What acceleration will a given torque produce in a car?**

Consider car below. Suppose it has a mass of mc = 1000kg. And suppose it has 4 wheels each with a radius r = 40cm, and mass mw = 20kg. If the engine exerts a torque of

τaxle = -500 Nm on the wheel (through the axle), how fast will the car accelerate down the road?

We will look at the forces acting on the wheels themselves. And since it should be the same for each wheel, we’ll look at just one wheel in particular. The forces acting on the wheel are shown below. There is a normal force from the ground. There is the force, Faxle, and torque, τaxle exerted on the wheel by the axle which itself is turned by the engine. The direction of the Faxle is unknown so I’ve just drawn it in some arbitrary direction. There is.a static friction force pointing to the right (since the torque, τaxle, exerted by the axle would try to turn the wheel so that the bottom surface of the wheel would slide to the left). Finally we have the force of gravity acting on the wheel itself.



In order to get the wheel to move (clockwise) we need the net torque on it so that,



where we use the fact that the wheel is approximately a disk so I = (1/2)mr2. But we don’t know what Fs is. In order to determine this, we use N2L of translation – on the entire car itself. Since there are 4 wheels, there are 4 forces of friction acting on the car. So we have:



Filling this into the torque equation we get,



Plugging these in…



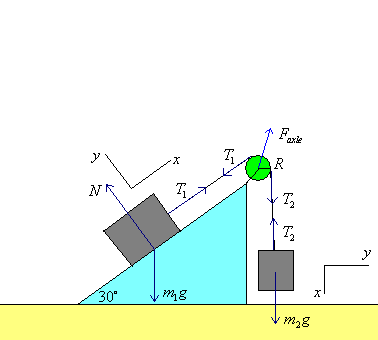
If the coefficient of static friction, μs = 1.2, what is the maximum acceleration that we can give the car?

Go back to N2L for the car itself.



**Example: Blocks on inclined plane connected to massful pulley**

Suppose we have two blocks, m1, and m2 connected to eachother, on an inclined plane. And suppose that we drape them over a cylindrical pulley, of mass M, radius R. What is the acceleration of the blocks? Take the axle in the pulley to be frictionless.



Before, when we analyzed this problem, we implicitly assumed that the pulley was negligible. But now we’ll take into account the full implications of the fact that it is rotating. Note that this requires the tension in the rope to be non-uniform. If the tension were uniform, i.e., T1 = T2 = T, then that would imply the pulley wouldn’t rotate since the net torque on it would cancel out. So we must assume the tensions are unequal. Also, we should realize that the pulley support exerts a force on it at the point of contact – i.e. at the center. Otherwise, labeling the forces proceeds as usual.

Once we’ve labeled the forces we can determine the acceleration. We can do this in either of two ways. First, we’ll apply N2L to each object individually and solve for a. Afterwards we’ll attempt to apply N2L to the system as a whole and calculate a. Apropos the first way, we have,



As for the pulley, using its rotation center as O, we can sum the torques acting on it. Note that the contact force Faxle, will not appear in the equation because it does not exert a torque about O. So we have,



In the second/third line we use the fact that a cylinder rotating about its middle has a moment of inertia I = (1/2)MR2, and that α = a/R. Now for the second mass we have,



These are our three equations, which we can solve for a, T1, and T2. We’ll content ourselves with a. So we solve for T1 using the first equation,



and solve for T2 using the last equation.



and plug both these into the middle equation (for the pulley)



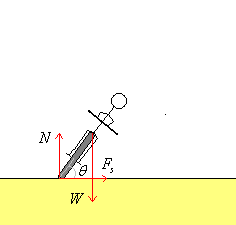
So our result is:



Observe how this result reduces to the earlier one if we set M = 0, i.e., if we ignore the pulley.

**Example: How far must lean over when making turn on your bike?**

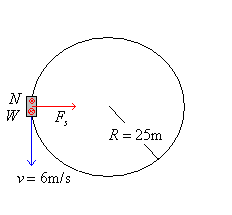
Suppose your riding your bike around a curve of radius R = 15m, and speed v = 10m/s, how far over, θ, must you lean your bike to keep from flipping over? Suppose the mass of you and the bike are 80kg.



There are three forces acting on you/bike. One is N, and Fs acting on the tire, and also gravity acting through the center of mass of you/bike. In order to keep from flipping over the torque acting about the center of mass of the person must be 0. So we must have,



Now don’t know what Fs is. So we use N2L for translation. Look at the forces from the top-down perspective, and since its circular motion, consider the forces along the centripetal, tangential, z directions.





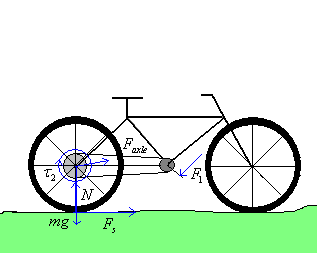
Filling Fs, N into the torque equation we have,



**Example: riding bike**

A common example of connected gears is your bike (or a car which has the same kind of set up – minus the chain). Instead of the gears being in physical contact though, they are are connected by a chain – but the result is the same. Consider the following example.

Suppose the bike pedal has a length ℓ = 12cm. And you step on it with a force of F1 = 300N perpendicular to the pedal. Suppose the pedal is connected to gear 1 (r1 = 5cm), which is connected to gear 2 (r2 = 10cm). And finally suppose the gear 2 is attached to the wheel (m = 10kg, R = 60cm). If the mass of you and the bike is a total of M = 90kg, how fast will you accelerate down the road? Assume the axle about which the wheel turns is frictionless.



First we draw all the forces acting on the wheel (back wheel). The wheel is connected to the axle, and so the axle will exert a force Faxle. We assume the axle is frictionless and so it will not exert a torque. The wheel is connected to gear 2, and so gear 2 will exert a torque on it, τ2. The wheel is in contact with the ground and so there will be normal and friction force on it. Finally the wheel will experience a gravitational force, mg. Now apply N2L (rotation) to the wheel itself. We’ll treat the wheel as a hoop since it is largely massless in the center. Now to determine the acceleration, first apply N2L to the bike/you itself



and now apply the torque equation to the rear wheel.



Now plug this into the first equation,

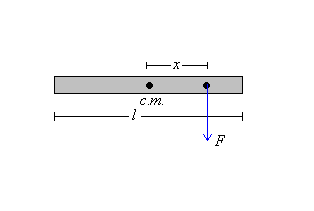


So what is τ2? From the equation in the previous example, it is just τ2 = (r2/r1)τ1. Now torque 1 is τ1 = -ℓF1sin90˚ = (0.12m)(300N) = -36N·m, and so the magnitude of torque 2 is: τ2 = (10cm/5cm)(30N·m) = 60N·m. Filling this in we get,



**Example: Center of percussion**

The center of percussion is the point on an object (usually a bat) which when struck gives the edge (or handle) a net acceleration of 0. For instance when a bat is hit outside the center of mass, then the bat will receive a torque from the ball which will cause it to rotate about its c.o.m., and thus cause the bat handle to rotate. Of course it will also receive a translational impulse causing the whole bat to move backward (including the handle). If it is hit in exactly the right spot – the center of percussion – then these will accelerations will cancel each other out and the handle will not move. Consider



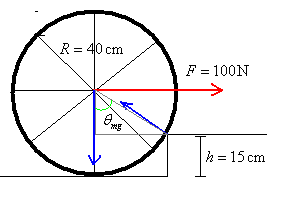
Suppose an object – a bat here – receives a force, F, at the point x. Where should x be so that the handle located at the left hand corner doesn’t get accelerated? The acceleration of the handle will come from two terms: the general acceleration of the center of mass, and the rotation about the center of mass. Consider then the net acceleration,



We want this to equal 0 so…



8. Suppose you pull on a wheel with the force as indicated If the wheel has a mass of 5kg, what is its angular acceleration about the step’s corner. Assume the wheel is just off the ground when you do this calculation.



As the wheel is off the ground, only 3 forces are exerted on it: F, mg, and the contact force at the corner. If we make our axis of rotation to be the corner, then only F and mg exert torques. The sum of these are:



where we’ve used the parallel axis theorem to get I about the corner. The angles are:

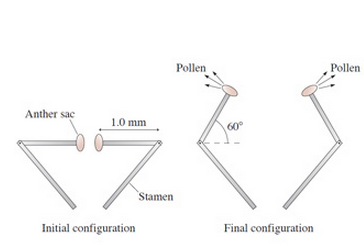


and so the net torque is:



**Problem**

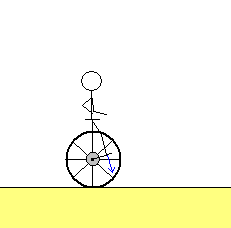
The bunchberry flower has the fastest-moving parts ever observed in a plant. Initially, the stamens are held by the petals in a bent position, storing elastic energy like a coiled spring. When the petals release, the tips of the stamen act like medieval catapults, flipping through a 60∘ angle in just 0.5ms to launch pollen from anther sacs at their ends. As the following figure shows, we can model the stamen tip as a 1.0-mm-long, 12 μg rigid rod with a 10 μg anther sac at the end. Although oversimplifying, we'll assume a constant angular acceleration. So how large is the straigtening torque?

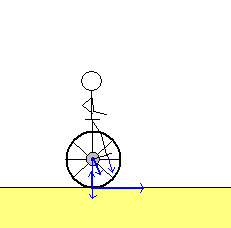


Moment of inertia of our object is: I = (1/3)Mℓ2 + mℓ2 = (1/3)(12×10-9kg)(0.001)2 + (10×10-9)(0.001)2 = 1.4 × 10-14. And the angular acceleration is θ = θ0 + ω0t + (1/2)αt2 → π/3 = (1/2)αt2 → α = 2π/3t2 = 8.38×106 rad/s2. And so the torque is τ = Iα = (1.4×10-14)(8.38 × 106) = 1.17×10-7 N∙m.

**Question 5**

Consider yourself on a unicycle. It, i.e. the wheel (which you may treat as a hoop), has a mass of 15kg, and radius 0.4m. You exert a force F = 150N perpendicular to the pedal (shown in blue), a distance of 25cm from the center of the axle. If you have a mass of 65kg, what resulting acceleration down the street will you have? Remember, you will have to look at the forces acting on just the wheel, and separately, the forces acting on you+wheel, as was done for the car.





Forces acting on wheel are static friction, the normal force, the force the axle exerts, the force that the pedal exerts via the axle, and gravity. Use the torque equation on the wheel.



Next, use the fact that α = a/r to write,



Next we have to look at the forces on the wheel+you, in the x-direction. So…

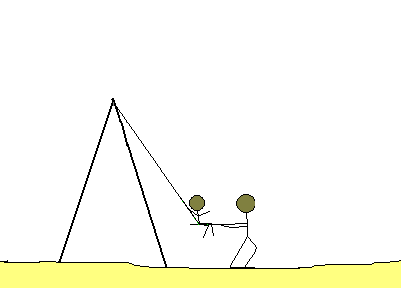


Plugging this into the torque equation…



**Problem 6**

Suppose you release a child on a swing as shown. The child returns 2.5s later. How long is the chain holding the swing?

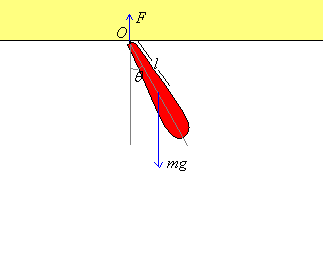


We can use the equation for the period of a pendulum,



**Example: Pendulum**

Consider an arbitrary shaped pendulum, attached to a ceiling say, and allowed to freely rotate. What will be its frequency of oscillation? Actually, let’s answer a more general question. If we release the pendulum from an initial angle θ0, how will its angle vary with time?



We can answer this question by applying the torque equation to it. Let I be its moment of inertia, and let ℓ be the distance between O and its center of mass. And let it have a mass m.

To determine the motion of the bat we must first identify all the forces acting on the bat. There are two. F is the force attaching the bat to the ceiling. The direction and magnitude of F probably varies with time. The other force is gravity, which will act through the center of mass at ℓ. Let’s consider the torques acting on the bat about the point O. Note that the force F doesn’t exert a torque about point O because its r is 0 about that point. So we only have,



Assuming only small angles of oscillation then we can replace sinθ with θ. This gives us,



The basic method of solving any differential equation is to postulate a functional form θ(t) containing some arbitrary parameters and then see if you can adjust the parameters to make θ(t) satisfy the differential equation and initial conditions. When you take differential equations, this is the first method they’ll talk about. Now luckily for us, we know what the solution should look like. Clearly θ will be oscillating back and forth, so θ(t) should be sinusoidal. Therefore we will postulate the following form,



where A, ω and φ are so far unknown constants. Now we see if we can choose A, ω, and φ in such a way to make θ satisfy the differential equation (and initial conditions). First let’s plug it into the ODE and see what we get,



so we have solved for the angular frequency. Now let’s determine A and φ. The initial condition is that θ(t=0) = θ0. Plugging our guess into this requirement we have,



another requirement is that we release the pendulum from rest. This means that . Plugging in our guess into this initial condition…



Looking back, this condition then implies that A = θ0. Therefore we have for our solution,:



Recalling some properties of these trig functions, we see that the period of oscillation of the pendulum is:



As a special case, consider the pendulum to be a mass on a string. Then I = mℓ2, and the formula for T simplifies to:



Interestingly then, the period

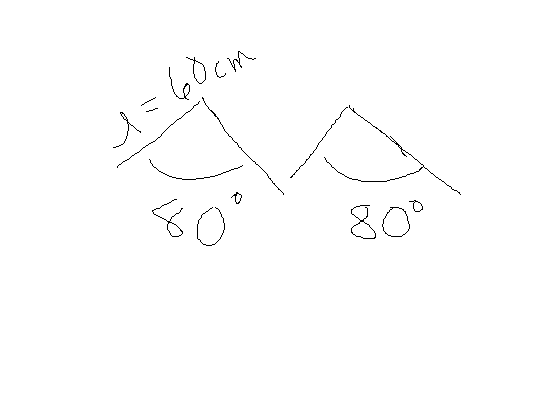
10. The wrecking ball on campus oscillates back and forth with a period of 6s. What is the length of the line holding the ball? E

Use the formula for the period of a pendulum,



**Example**

Suppose a monkey swings from tree to tree like this below. How fast would he go?



So time it takes is ½ a period so t = π√ℓ/g. And then distance covered is Δx = 2ℓsin(40). So ratio is the velocity.

**Example**

Suppose a child of mass m =30kg oscillates back and forth on a swing of length ℓ = 2m. Her amplitude decreases by factor of 1/2 in time 20s. What is the damping coefficient? What is her period?



And so period is T = 2π/ω = 3s.

Then suppose you push on the swing with a periodic force F(t) at the resonant frequency and force max F = 100N. What amplitude will the oscillations build up to? Well we’ll have:



2. Orangutans move by brachiation, swinging from tree to tree. Suppose we can model an orangutan as a uniform orang-a-board of length 2.1m. And suppose that it starts and ends each swing at a 30° with respect to the vertical. What will be its average horizontal velocity?

The length of time required for the Orangutan to travel from one end of the swing to the other is half a period. So



And the horizontal distance traveled will be:

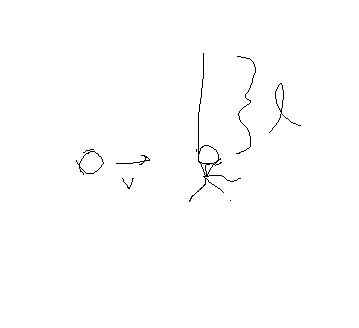


So the average velocity will be:



**Example**

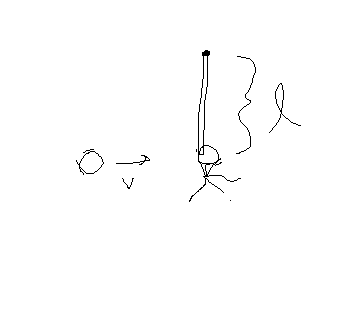
A monkey (m = 10kg) hangs on a massless vine of length ℓ = 5m. You throw a ball (m = 0.5kg) horizontally at speed v = 18m/s, which the monkey catches. Supposing the catch happens instantaneously, what will be the angular position of the monkey thereafter as a function of time?



θ(t) = Acos(ωt + φ0). ω = √(g/ℓ) = 1.4 rad/s. Initial angular velocity follows from conservation of angular momentum: ω0 = Iballωball/(Iball + Imonkey) = mballℓ2vball/ℓ / (mballℓ2 + mmonkeyℓ2) = mballvball/(mball + mmonkey)ℓ = 0.171 rad/s. And initial position is: θ0 = 0. So then we have θ(t) = Acos(1.4t+φ0). And (1) 0 = Acos(φ0), (2) 0.171 = -Aωsin(φ0) → -0.122 = Asin(φ0). So A = 0.122, and φ0 = -π/2. And therefore θ(t) = 0.122cos(1.4t – π/2).

**Example**

A monkey (m = 10kg) hangs on a vine of length ℓ = 5m and mass m = 20kg (can treat as a uniform board). You throw a ball (m = 0.5kg) horizontally at speed v = 18m/s, which the monkey catches. Supposing the catch happens instantaneously, what will be the angular position of the monkey thereafter as a function of time?

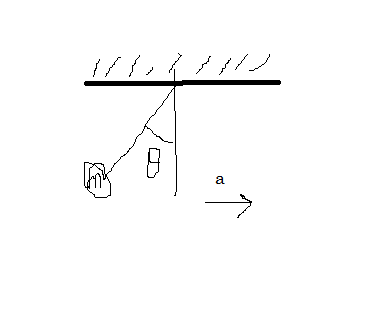


θ(t) = Acos(ωt + φ0). position of center of mass would be ℓ = [(20)(2.5) + (10)(5) + (0.5)(5)]/(10+20+0.5) = 3.36m.

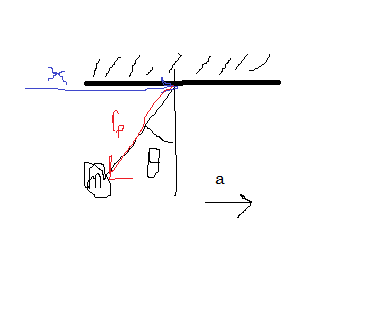
ω = √(mgℓ/I) = 1.53 rad/s. Initial angular velocity follows from conservation of angular momentum: ω0 = Iballωball/(Iball + Imonkey + Ivine) = 0.105 rad/s. And initial position is: θ0 = 0. So then we have θ(t) = Acos(1.53t+φ0). And (1) 0 = Acos(φ0), (2) 0.105 = -Aωsin(φ0) → -0.069 = Asin(φ0). So A = 0.-69, and φ0 = -π/2. And therefore θ(t) = 0.069cos(1.53t – π/2).

**Example**

Suppose you’re driving along in a car, accelerating at rate acar. What will oscillation period of pendulum, of length ℓ, be now?



Well, can write position of pendulum as **r** = **x**i + **r**p, where **r**p is position about pivot point.



N2L for translation reads:



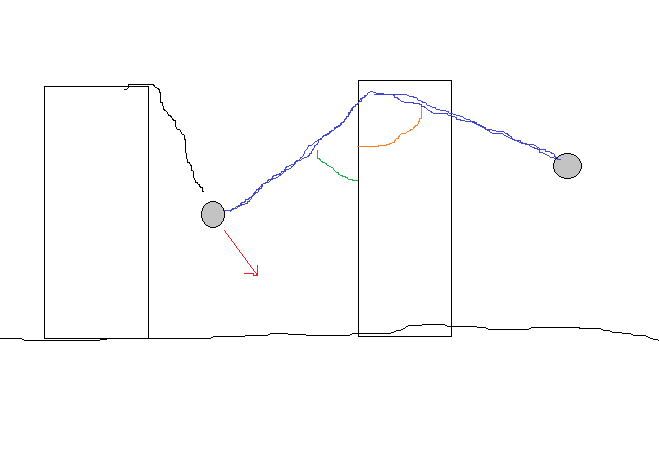
Suppose I cross both sides of the equation by **r**p. Then ….



Hmmmmm.

**Spiderman problem**

Spiderman jumps off building and shoots web (ℓ = 50m) going at speed v = 30m/s, at initial angle of 30 degrees. What max angle will he reach and when.



So period is T = 2π/ω; ω = √mgℓ/I = √ℓ/g. And then have to solve θ(0) = Acos(φ0) → -π/6 = Acos(φ0), and ω(0) = -Aωsin(φ0) → 30/50 = -Aωsin(φ0). Get A usual way, and get φ0 via: φ0 = tan-1(-stuff/-stuff). So note angle is in the third quadrant. So answer is θ(t) = Acos(ωt – stuff) and so we have A and t (from ωt – stuff = 0).

**Walking speed**

Suppose your leg rotates from -30 to 30 degrees when walking, and is 0.8m long. What would be a typical period? Δt = T/2 = π√(I/mgℓ) = π√(0.33mℓ2/mgℓ) = π√(0.33ℓ/g) = 0.52s. And what horizontal distance will have been covered? Δx = 2∙ℓsin(30) = ℓ = 0.8m. And so distance traveled would be Δx/Δt = 0.8m/0.52s = 1.54m/s.

**Question 2.** A nerdy babysitter encourages a child (mass = 20kg) to leap into a swing seat (mass = 5kg and initially stationary) with a speed of 4m/s. The swing chain (assumed to be massless) is 3m long. What will the babysitter infer to be the child’s angle as a function of time?

The initial angle of the child/seat will be θ0 = 0°. From momentum conservation we can find the velocity of the child/seat when they collide (20kg)(4m/s) + (5kg)(0m/s) = (25kg)v 🡪 v = 80/25 = 3.2m/s. And this corresponds to an angular velocity of ω0 = v/r = 3.2/3 = 1.07rad/s. The angular frequency of motion is ω = √(mgℓ/I) = √(ℓ/g) = √(3/9.8) = 0.553 rad/s. So the angle will be generically given by θ(t) = Acos(0.553t+φ0). Plugging in the initial conditions we have 0 = Acos(0.553∙0+φ0) 🡪 0 = Acos(φ0) 🡪 φ0 = -π/2. And initial velocity conditions require: 1.07 = -Aωsin(0.553∙0-π/2) 🡪 A = 1.07/ω = 1.07/0.553 = 1.94 rad. So θ(t) = 1.94cos(0.553t-π/2).